Knowledge Assessment App (KAA)

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Introduction

The KAA is a Shiny App developed by the University of Padua within the Erasmus+ TquanT Project. Its aim is to give a demonstration of how the adaptive assessment of knowledge developed within the Knowledge Space Theory (KST) framework works in practice. The report is organized as follows: after the introduction, the second section describes, briefly, the main concepts of the KST; the third section presents the basic elements of assessment of individual knowledge and the fourth section shows how to use the KAA Shiny App. Finally, in the last section some exercises can be found, in order to guide the use of the shiny app.

Knowledge Space Theory

KST (Doignon & Falmagne, 1985, 1999; Falmagne & Doignon, 2010) is a mathematical theory developed by Jean-Paul Doignon and Jean-Claude Falmagne with the aim of assessing individual knowledge in specific fields. Psychometric theories (e.g., classic test theory) usually assess **"how much a student knows"** about a certain field, producing a numerical score. Instead, KST tries to describe **"what a student knows"** about that field.

In KST, the students' knowledge is represented by the subset K of all the problems of a given domain Q that a student is capable of solving. The set Q is named *knowledge domain* and the subset K is named *knowledge state*. The population of students is represented by a *knowledge structure*, which is a pair $(Q, \mathcal{K})$, where $\mathcal{K}$ is any collection of knowledge states which contains at least Q and the empty set.

In KST, the objective of an assessment consists in assigning to each student the right knowledge state, out of those contained in the knowledge structure, asking as few questions as possible. The theory is applied for the adaptive assessment in different fields; from the school (e.g. ALEKS system, www.aleks.com) and the university (stat-knowlab.unipd.it, http://stat-knowlab.unipd.it) education, to the assessment in clinical psychology (Spoto, Stefanutti, & Vidotto, 2010; Spoto, Bottesi, Sanavio, & Vidotto, 2013).

Adaptive Assessment in KST

The procedure for adaptively assessing is based on the assumption that not all the subsets of Q are contained in the knowledge structure $\mathcal{K}$. In fact, in real situations, dependencies between problems usually reduce the number of the admissible knowledge states (Falmagne & Doignon, 2010; Heller & Repitsch, 2012).

KST is characterized by two main components. The *deterministic* component uses discrete mathematics to build a theoretically plausible structure. In deterministic assessment, the answers to properly selected problems lead to identify a **unique knowledge state**. In the real world situation students could give a correct answer to a problem by chance (*lucky guess*) or an incorrect answer by distraction (*careless error*). The deterministic assessment excludes these possibilities and for this reason it does not represent an appropriate tool for assessing the state of a student.

In the *probabilistic* component of the theory a likelihood function associated with the knowledge states is continuously updated depending on the answers (BLIM; Falmagne & Doignon, 1988). This procedure is replicate until a large portion of likelihood is concentrated in a single state.

The assessment starts with prior information about the student which leads to define an initial likelihood distribution $\mathcal{L}_0$. If this prior information is missing or uncertain then the uniform distribution on the set of states is the best initial point(Heller & Repitsch, 2012; Hockemeyer, 2002). Starting from $\mathcal{L}_0$ the assessment is carried out iteratively. The probabilistic assessment procedure consists in the application of the following three rules:

1 **Questioning rule**: in each iteration n of the procedure, the first step consists in finding the most efficient question, based on the likelihood $\mathcal{L}_n$. The *half split rule* (equation (1)) is based on the idea that the most informative question q is the one that partitions the structure in two subsets $\mathcal{K}_q = \{K \in \mathcal{K} : q \in K\}$ and $\mathcal{K}_{\bar{q}} = \{K \in \mathcal{K} : q \in Q \setminus K\}$ having sizes as similar as possible.
So that, for each $K \in \mathcal{K}$, $\mathcal{L}_n(K) \geq 0$ is the likelihood that, at the step n, the student is in the knowldge state K.

$$\mathcal{L}_n(\mathcal{K}_q) = \sum_{K \in \mathcal{K}_q} \mathcal{L}_n(K).$$

of the states containing q is as close as possible to the one not containing q. The question q for which the following quantity is the smallest is chosen:

$$\min |2 \cdot \mathcal{L}_n(\mathcal{K}_q) - 1| \tag{1}$$

If more than one question satisfy this condition, then one of them is selected at random.

From a psychological perspective, the half split rule selects questions that are neither too difficult and demotivating nor too easy and consequently too boring for the student.

2 **Updating rule**: this rule establishes how the likelihood distribution changes as a function of the answer to question q. The *multiplicative updating rule* increases (decreases) the likelihood of all the states containing q and decreases (increases) the likelihood of all the states not containing q if a correct (incorrect) answer is observed to q. It is defined as follows: for $K \in \mathcal{K}$,

$$\mathcal{L}_{n+1}(K) = \frac{\pi_K^q \mathcal{L}_n(K)}{\sum_{K' \in \mathcal{K}} \pi_{K'}^q \mathcal{L}_n(K')} \tag{2}$$

In equation (2) the likelihood of the states for the step $n+1$ depends on the parameter π_K^q which is a conditional probability of the observed response to problem q depending on two parameters: a probability β_q of a careless error and a probability η_q of a lucky guess. Then π_K^q is defined by:

$$\pi_K^q = \begin{cases} \beta_q & \text{response incorrect and } q \in K \\ 1-\eta_q & \text{response incorrect and } q \notin K \\ 1-\beta_q & \text{response correct and } q \in K \\ \eta_q & \text{response correct and } q \notin K \end{cases} \quad (3)$$

When $\eta_q = 0$ and $\beta_q = 0$ for all problems $q \in Q$ the assessment is deterministic.

3 **Stopping rule**: This rule specifies the criterion C that has to be met in order to terminate the assessment. The C criterion is a number between 0 and 1, and the assessment terminates at iteration n whenever the condition

$$\mathcal{L}_n(K) \geq C, \;\; K \in \mathcal{K}$$

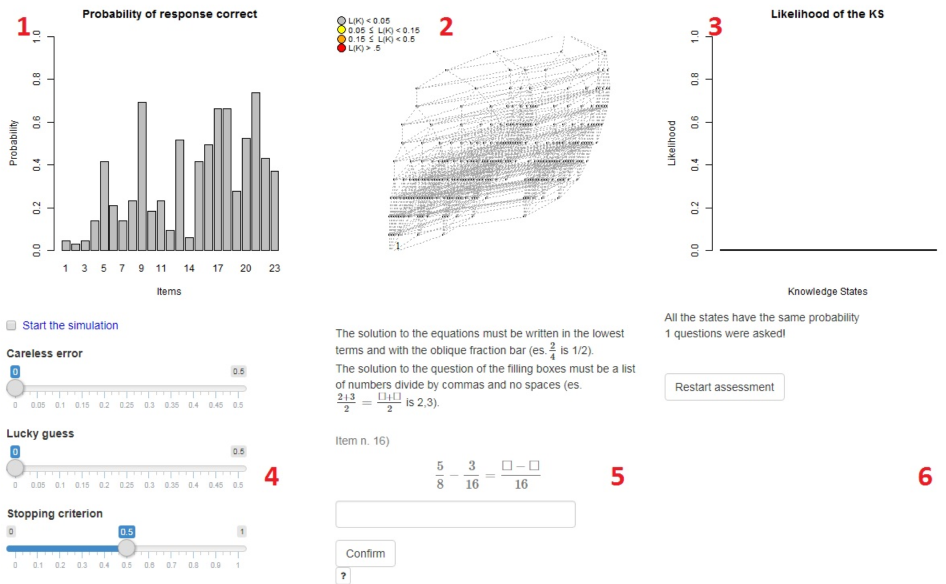
is satisfied. The C criterion should be sufficiently high to avoid more than a single modal state (recall that a state $K \in \mathcal{K}$ is modal at iteration n if its likelihood $\mathcal{L}_n(K)$ is maximum). See the Exercises Section in this respect.

Guide for users

The KAA Shiny App is developed to give a demostration of how an assessment in KST works in practice, by manipulating the β and η items' parameters, the stopping criterion C and monitoring the likelihood $\mathcal{L}_n$ of the knowledge states during the assessment. The App consists of the three pages: *Introduction*, *A Short Guide* and *Assessment*.

The *introduction* page contains a short presentation text on KST; the *A short guide* page contains a brief summary of this section with the basic instructions to use the *Assessment* page.

The assessment page uses a knowledge structure with 260 knowledge states. The example structure was built from 23 question that assessed the knowledge of fractions (Stefanutti & de Chiusole, 2017). The *assessment* page consists of two rows of elements. The top row contains diagrams that will be monitored during the assessment. The bottom row contains some controls for changing the parameter values, the problem that is currently presented to the student, and the intermediate or final results of the assessment.



The red numbers, from 1 to 6, in the screenshot allow to distinguish the different sections of the panel. A description of each section follows.

1 For each item $q \in Q$, this bar-graph represents the likelihood $\mathcal{L}_n(\mathcal{K}_q)$ that the problem q belongs to the student's knowledge state. This likelihood can be regarded as how likely it is that the student masters the problem. During the assessment, the probabilities $\mathcal{L}_n(\mathcal{K}_q)$ changes depending on the response provided by the student.

2 This element is the Hasse diagram of the knowledge structure and it represents the knowledge states in a quasi-order relation: from the smallest state (bottom of the diagram) to the largest (top of the diagram). During the assessment, the nodes representing the states may change in size [1]; moreover, the color of the states may change depending on the likelihood $\mathcal{L}_n(K)$ of the state K. More in detail, for values of $\mathcal{L}_n(K) \leq .05$ the states turn gray, for value of $\mathcal{L}_n(K)$ between .05 and .15 the states turn yellow, for value of $\mathcal{L}_n(K)$ between .15 and .5 the states turn orange and for values of $\mathcal{L}_n(K) \geq .5$ the states turn red.

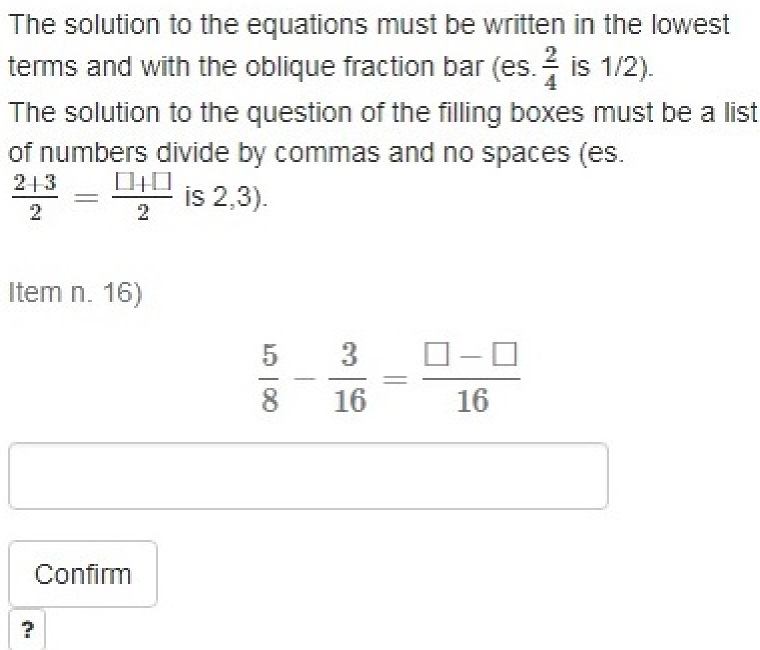
3 This histogram represents the likelihood distribution $\mathcal{L}_n(K)$. Provided that the lucky guess and careless error parameters are not too high, after a reasonable number of iterations the most part of the likelihood should be located on a single state.

4 These controls (three sliders and a check button) allow the tune of three parameters of the assessment procedure. The sliders of the *lucky guess* and the *careless error* probabilities lead to manipulate the amount of noise in the assessment. In order to simplify the procedure, it is assumed that all the lucky guess and careless error probabilities are equal for all the problems (in real situations this may not be the case).

[1] The size allows a comparison only among states in the same iteration n

The slider of the *stopping criterion* lead to manipulate the minimum value that the likelihood of a knowledge state must have to stop the assessment procedure.

5 In this section appears the problem $q \in Q$ that the assessment procedure selects in each iteration. In the white form the user must write the response and press the button *Confirm* (see the picture below). The procedure compares the user's answer with the correct answer and updates the others figures and informations. The problems used in KAA belong to the knowledge domain of fractions and were used in Stefanutti and de Chiusole (2017).

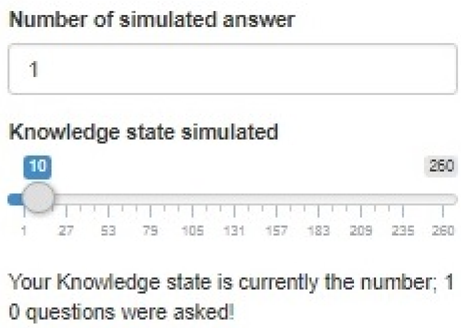


6 In this section the main output of the assessment is displayed. The states with maximum likelihood along with the number of questions that have been asked so far are shown. When the assessment is over,

the list of problems that the student is able to solve (i.e., belonging to the student's knowledge state) is reported.

When the user is done with the interactive modality, the simulation modality can be experimented by the checking button.

In the simulation modality, it is possible to select a state of knowledge among the admissible 260 as the *"student true knowledge state"* using the slider show in the figure below. The *Procede with simulation* produce a simulated response coherent with the selected state. During the simulation modality is possible to modify the parameters and see how the assessment convergency will be altered. To make the simulation faster the user could choose a larger number of simulation steps for each click. This modality is helpful in assessment with higher noise level, because in this case the number of iterations could be quite large.



Exercises

- After a few iterations and before the assessment terminates, have a look of the bar diagram in section (1)of the assessment page. Are there problems that belong in all certainty to the student's knowledge state? Are there problems that do not belong in all certainty to the student's state.
- What is the minimum Stopping criterion for obtaining a unique state with maximum likelihood? Why?
- Looking again at the diagram in section (1) it can be noticed that at the setup, the likelihood of the problems does not resemble to a uniform distribution. Why? (Hint: this likelihood is obtained as a function of the likelihood of the states).
- Is it possible for a problem to be presented twice? Did it ever happen? If so, in what situation did it occur? Did you have look at the likelihoods of the problems?
- How the stopping criterion modifies the results of the assessment?
- The first question asked by the procedure is always the number 16, why does it happen?
- Sometimes the procedure does not converge into a knowledge state, and the likelihood distribution appears to be rather unstable in going from one iteration to the next. Why does this happen? Is it related to the values of certain parameters of the procedure? Which ones?

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